MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

DMT5131 – MATHEMATICAL TECHNIQUES 1

(For DIT students only)

14 OCTOBER 2019
02.30 pm – 04.30 pm
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **4** pages (**2** pages with **3** questions and **2** pages of Appendix). Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

QUESTION 1

a) Solve the equation of $\sqrt{7x+9} = x+3$. (2 marks)

b) Solve the inequality $\dfrac{x-4}{(x+2)(2x+1)} < 0$ and express your answer in interval notation. (3 marks)

c) Given a quadratic function of $f(x) = 2(x-2)^2 - 18$.

i) State the vertex of the function. (0.5 mark)

ii) Find the coordinates of x-intercepts. (3 marks)

iii) Find the coordinate of y-intercept. (1.5 marks)

[TOTAL 10 MARKS]

QUESTION 2

a) Find matrix Q if $4Q + \begin{bmatrix} 2 & -1 & 0 \\ 4 & 7 & -3 \end{bmatrix}^T = \begin{bmatrix} 10 & 12 \\ -3 & -1 \\ 4 & 9 \end{bmatrix}$ (2.5 marks)

b) Find value of x for which the following matrix is not invertible. (2 marks)

$$\begin{bmatrix} x-2 & 2 \\ x & 3 \end{bmatrix}$$

c) Jenny enjoys making desserts for her neighbours. Last month, she made 3 batches of cream puff cake and 5 batches of cookies, which used a total of 22 eggs. The month before, she baked 2 batches of cream puff cake and 2 batches of cookies, which required a total of 12 eggs.

i) Represent the above information in the form of $\boldsymbol{AX} = \boldsymbol{B}$. (1.5 marks)

ii) Find A^{-1}. (2.5 marks)

d) The matrix given below is in the form of $\boldsymbol{AX} = \boldsymbol{B}$.

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & -5 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -28 \\ 24 \end{bmatrix}$$

i) From the information given, write down a system of linear equation. (1.5 marks)

ii) Use Cramer's Rule to solve for y. (5 marks)

[TOTAL 15 MARKS]

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QUESTION 3

a) Given the first term of the sequence is -3. List the first four terms for the general sequence form of $a_n = \frac{a_{n-1}+4}{2^n}$. (2.5 marks)

b) Sally played an adventure game named Catching Dragon. She scored 300 points for capturing her first dragon and then 700 points for capturing her n^{th} dragon. While the total number of points for capturing all n dragon was 8500.

i) Given that the number of points that Sally scored for capturing each successive dragon formed an arithmetic sequence, find the value of n. (2 marks)

ii) Find the difference, d for each dragon captured. (2 marks)

c) Given that the 3rd term of geometric sequence is $-\frac{9}{4}$ and common ratio is $\frac{3}{4}$.

i) Find the 1st term. (1.5 marks)

ii) Find the sum of 13 terms. Leave your answer in 4 decimal places. (1.5 marks)

iii) Evaluate the sum to infinity. (1 mark)

d) Expand $(2x-3y)^3$ using the Binomial Theorem. (4.5 marks)

[TOTAL 15 MARKS]

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APPENDIX – KEY FORMULA

Completing the square: $x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$

Quadratic formula: If $ax^2+bx+c=0$ where $a\neq 0$, then, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Standard form of a quadratic function: $f(x)=a(x-h)^2+k,\ a\neq 0$

Determinant of a 2 × 2 matrix	*Determinant of a 3 × 3 matrix*
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=a_1b_2-a_2b_1$	$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $=a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}-b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}+c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

Inverse of a 2 × 2 matrix
If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad-bc\neq 0$.

Cramer's Rule for 2 × 2 matrix	*Cramer's Rule for 3 × 3 matrix*
If $\begin{matrix} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{matrix}$ then $x=\dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ and $y=\dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$, where $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}\neq 0$	If $\begin{matrix} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{matrix}$ then $x=\frac{D_x}{D},\ y=\frac{D_y}{D},\ z=\frac{D_z}{D}$ where $D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, $D_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, $D_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

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Arithmetic sequence	*Geometric sequence*
$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}$, $S_n = \dfrac{a_1(1-r^n)}{1-r}$
$S_n = \dfrac{n}{2}(a_1 + a_n)$	$S_\infty = \dfrac{a_1}{1-r}, \lvert r \rvert < 1$

Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k; \quad n \geq 1$$

The r^{th} term of the expansion of $(a+b)^n$ is $\binom{n}{r-1} a^{n-r+1} b^{r-1}$.

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